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Topology optimization of thermoelectric generator for maximum power efficiency

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Abstract

Thermoelectric generators offer a promising approach for harvesting waste heat from both natural and human-made sources, enabling sustainable electricity generation. While geometric design plays a crucial role in optimizing device performance, conventional approaches remain confined to simple configurations, limiting efficiency improvements. This constraint arises from the complex interplay of multiphysical interactions and diverse thermal environments, which complicates structural optimization. Here, we introduce a universal design framework that integrates topology optimization (TO) with additive manufacturing to systematically derive high-efficiency thermoelectric 3D architectures. By formulating an optimization problem to maximize power generation efficiency, our approach explores an unprecedentedly large design space, optimizing the geometries of thermoelectric materials across diverse thermal boundary conditions and material properties. The resulting TO-derived geometries consistently outperform conventional cuboids, demonstrating significant efficiency gains. Beyond *in-silico* studies, we provide theoretical insights and experimental validation, confirming the feasibility of our design approach. Our study offers a transformative way for enhancing thermoelectric power generation, with broad implications for next-generation sustainable energy technologies.

Main

Geometry plays a fundamental role in functional performance in both natural and engineered systems. In nature, evolutionary processes have shaped optimized architectures that exhibit exceptional properties. For instance, nacre in mollusk shells balances strength and toughness through its layered structures¹, while gecko footpads feature microscopic geometries that enhance adhesion via van der Waals forces². Similarly, geometric design is equally critical in materials engineering, where structural functionalities are often optimized through shape modifications. Examples include lattice architectures for mechanical stiffness³, photonic crystals for light manipulation⁴, and heat sink designs for thermal management⁵. Despite its significance, however, geometry is often overlooked in engineering design, which traditionally relies on empirical heuristics or trial-and-error methods—an approach that limits the exploration of unconventional yet efficient geometries. Recently, topology optimization (TO) fundamentally challenges this oversight by systematically generating geometries that maximize performance beyond conventional design constraints⁶. Unlike traditional approaches, TO redistributes material freely, unlocking unprecedented design flexibility and efficiency gains⁷. This approach has demonstrated success across aerospace⁸, automotive⁹, and materials science¹⁰. When combined with advanced manufacturing techniques such as three-dimensional (3D) printing.¹¹⁻¹³, TO further expand design possibilities and facilitate the fabrication of complex, high-performance structures that were previously impractical or impossible.

Thermoelectric (TE) energy conversion has drawn significant attention due to its ability to directly convert heat into electricity, offering a sustainable approach to harvesting waste heat from natural and human-made sources, including residential, industrial, and transportation systems¹⁴⁻¹⁸. Like many other engineering systems, TE generators (TEGs) demonstrate the

importance of geometric design, as their performance can be enhanced not only through material advancements but also through the change of their geometry¹⁹⁻²⁴. Properly engineered TE material geometries within a TEG can optimize heat flow, enhance thermal gradients, and improve electrical output, ultimately leading to substantial performance gains. However, the inherent complexity of TE systems—stemming from the intricate interplay between thermal and electrical transport, as well as varying thermal environments—presents significant design challenges²⁵⁻²⁷. Current design strategies, which rely predominantly on empirical knowledge and heuristic-based modifications, remain confined to a narrow design space, resulting in simple shapes and topologies with limited performance improvements.

Here, we introduce a TO framework for the design of TE materials aimed at maximizing power generation efficiency. As illustrated in Figure 1, our methodology employs finite element-based TO to generate geometries that align with specified objective functions by incorporating system parameters as inputs. Compared to cuboidal TEGs, the optimized geometries consistently exhibit superior power generation performances across all tested boundary conditions and materials, achieving enhancements of up to eightfold. The experimental results obtained from 3D-printed materials closely aligned with computational predictions, confirming the validity and feasibility of our approach. These findings establish TO as a powerful platform for designing high-performance TE devices with broad applicability in real-world energy harvesting systems.

Optimums with different thermal boundaries

We applied TO on a single-leg TEG sandwiched between top and bottom copper electrodes to derive optimal leg geometries that maximize power generation efficiency (η). The overall

optimization process is illustrated in Fig. 1b. Extrinsic thermal boundary conditions are a crucial factor influencing the overall power generation performance of a device^{19,28-30}. Accordingly, we optimized the previously reported 3D-printed BiSbTe model material under various boundary conditions, incorporating convection, heat flux, and temperature (Fig. 2 and Supplementary Fig. 1)³¹. Initially, optimization was conducted under convective heating and cooling, with the convection coefficient (h) varied from 10 to 1000 W m⁻² K⁻¹ (Fig. 2a and Supplementary Fig. 2), covering the wide range of thermal environments encountered in TEGs. The optimized geometries exhibit broadly similar features, primarily adopting an I-shape. Under all conditions, the optimal geometries achieved higher η than both the initial model and the cuboidal models of equivalent volume. As h decreased on both sides, the refined geometric features became thinner, and the overall relative volume of the optimized geometry gradually decreased—resulting in a larger enhancement in η relative to cuboidal models. The maximum η enhancement of 79% was observed under the strongest cooling and weakest heating conditions (Fig. 2h, l).

To better reflect heat transfer conditions commonly encountered in industrial and practical engineering applications, we further applied the TO process under three specific boundary conditions. First, we considered a fixed heat-flux (Q_h) combined with convective cooling (Fig. 2b, c and Supplementary Fig. 3). This boundary condition is widely recognized as the most common thermal environment in practical applications, particularly when a TEG is applied to heat sources, with the cold side being cooled via passive or active cooling mechanisms³²⁻³⁵. Similar to the case under convective heating and cooling, the optimized geometries exhibit pronounced lateral trimming, yielding an I-shape. As the Q_h decreases, the refined geometric features become thinner while the overall volume gradually contracts. At the Q_h of 5000 W·m⁻², the optimized geometry achieves ~59% enhancement in η compared to a cuboid (Fig. 2i).

As the second condition, we examined the cases where the hot-side temperature (T_h) is fixed, and a constant heat flux (Q_c) is applied at the cold side. This setup represents the operating condition of a hot reservoir with systemized cooling^{32,36}. Under these conditions, the optimal geometry takes the form of an asymmetric hourglass, with a greater proportion of material near the lower part (Fig. 2d, e and Supplementary Fig. 3). Each optimized design is cooled to the preset temperature limit (Supplementary Fig. 4), indicating that the TE material adopts an hourglass to maximize temperature difference (ΔT). The asymmetry between the top and bottom likely results from the redistribution of material to the lower-temperature region, where the material exhibits a high figure-of-merit (ZT). As the Q_c increases, the optimal geometry requires a larger material volume. At the Q_c of $5000 \text{ W} \cdot \text{m}^{-2}$, the optimized geometry achieves ~64% enhancement in η compared to a cuboid (Fig. 2j).

The third boundary condition we investigated involves fixing both hot-side and cold-side temperatures (T_c) (Fixed- T conditions). This setup is widely used in laboratory experiments to evaluate the power generation of TE devices. However, it assumes continuous and infinite heat transfer at both junctions, making it relevant only to idealized scenarios such as hot and cold reservoirs^{19,37-39}. Under this condition, the optimized design maximizes the contact area with electrodes while maintaining the material volume below the preset maximum, resulting in only minimal side-surface trimming (Fig. 2f, g and Supplementary Fig. 3). This small geometric change is expected because the fixed-temperature boundary inherently determines the ΔT and the resulting electromotive force ($\mathcal{E}$) of the TE legs, leaving little room for optimization. Nonetheless, compared to a cuboid, the optimized geometries still achieve up to a 6% enhancement in η , primarily due to higher thermal resistance (R_{th}), which reduces the required heat flux Q_h (Fig. 2k).

Fundamentally, the TEG η is determined by the conversion factor ($\mathcal{E}^2/Q_h$) and the matching conductance (R_m/R_{el}),

$$\eta = \frac{\mathcal{E}^2 R_m}{Q_h R_{el}}$$

where $\mathcal{E}$ is given by $S\Delta T = SQ_h R_{th}$, where S is the Seebeck coefficient of materials, and R_{el} is the electrical resistance of the TEG. Additionally, R_m is the resistance matching coefficient, defined as $R_m = m/(1+m)^2$, where m represents the ratio of the load resistance to R_{el} . With thinner optimal geometries, an increase in R_{el} reduces η due to a decrease in matching conductance, whereas an increase in R_{th} enhances the conversion factor, leading to a higher η . Furthermore, the geometry can evolve so that the R_m approaches the optimum value of 1/4. Depending on the boundary conditions, TO intelligently balances these trade-offs, optimizing either the conversion factor or the matching conductance to improve overall η (Fig. 2m–o). For example, under weak heating or cooling conditions, TO conceives the thinner geometries to enhance R_{th} and maximize the conversion factor. Conversely, under strong heating or cooling conditions, TO derives thicker geometries to improve matching conductance, as larger ΔT values are naturally created by the boundary conditions (Supplementary Figs. 2 and 3).

The geometric evolution was further influenced by the temperature dependence of material properties. For example, in our model, BiSbTe exhibits a steeper temperature dependence in electrical conductivity (σ) than in thermal conductivity (κ) at low temperatures. This behaviour is also observed in typical TE materials, which exhibit the characteristics of heavily doped semiconductors. Accordingly, changes in R_{el} or the operating temperature range due to geometric evolution were more pronounced than changes in R_{th} . This relation explains the greater η enhancement under weak heating and strong cooling conditions, which shift the

operating temperature range to lower values, where the η gain from improved matching conductance outweighs the loss from a reduced conversion factor. Additionally, we examined how variations in the objective function, aspect ratio, load resistance, and contact resistance affect the optimal design (Supplementary discussion and Supplementary Fig. 5).

Optimums with different materials

We next extended the TO process to a diverse range of materials. Specifically, we incorporated nine different reported material properties into our optimization process under fixed Q_h and convective cooling^{19,21,31,40-45}. The selection of these materials was based on their operational temperature ranges, categorized into three groups according to their peak ZT values: low-temperature (300–500 K), mid-temperature (500–700 K), and high-temperature (>700 K) materials (Supplementary Fig. 6a). The optimization process consistently produced an I-shaped configuration across materials, with the final dimensions and proportions varying based on their thermal and electrical properties (Fig. 3a).

Among these, FeNbSb and PbTe exhibited notably high η enhancements, primarily due to their significantly larger ΔT relative to a cuboid and their steep temperature dependence in material properties (Fig. 3b and Supplementary Fig. 6). For instance, the optimal design with PbTe resulted in a 5942% increase in the conversion factor compared to a cuboid (Fig. 3c). This enhancement arose from the larger ΔT and relatively higher S at elevated temperatures—for example, $47 \mu\text{V K}^{-1}$ at 300 K and $236 \mu\text{V K}^{-1}$ at 700 K. Meanwhile, the optimal design with FeNbSb increased the conversion factor by 3672% due to its enhanced R_{th} , although the matching conductance was reduced by 67%, leading to a more than tenfold increase in η . In contrast, the optimum design with BiSbTe, AgCuTe, MgAgSb, and Cu₂Se—each exhibiting

weaker temperature dependence—showed smaller η enhancements. Nevertheless, η improvements ranging from 9% to 180% were observed in these materials, validating the universal applicability of geometric design for enhancing power generation. Additionally, we performed the same optimization under fixed- T conditions, where efficiency improvements exceeding 200% were achieved (Supplementary discussion and Supplementary Fig. 7).

To gain further insight into how material properties influence optimal geometries, we conducted TO on hypothetical materials by independently varying their κ , σ , and S (Supplementary Fig. 8). Regardless of material properties, TO consistently generated thinner optimal geometries, achieving enhancement in η compared to the cuboid model (Fig. 3d). This enhancement arises from an increase in the conversion factor, driven by a reduction in R_{th} and a corresponding increase in ΔT relative to cuboid models. Specifically, higher κ leads to thinner optimal geometries to sustain larger ΔT . Meanwhile, σ presents a more complex effect: designs with lower σ tend to develop a thicker central region, effectively reducing R_{th} through intelligent geometric shaping. In contrast, a lower S results in an optimized geometry with a thinner height at the top and bottom sections. Overall, these findings suggest that TO effectively balances the trade-off between the conversion factor and matching conductance to maximize η . For instance, a material with low σ can compensate by increasing its cross-sectional area, whereas materials with high κ or low S evolve into thinner geometries to enhance $\mathcal{E}$.

Integrated optimization and experimental validation

Building on these findings, we conducted an integrated optimization for three materials—BiSbTe, PbTe, and Cu₂Se—by incorporating the measured contact resistance values and adjusting the boundary conditions to match their respective properties under fixed Q_{h} and

convective cooling (Fig. 4). The resulting optimized geometries exhibited greater diversity than those obtained in previous cases. In particular, the optimal geometry for PbTe featured a large overall material volume with a distinct thin neck in the mid-lower region, while the optimal geometry for Cu₂Se formed an asymmetric hourglass shape, with more material concentrated near the hot side. To experimentally validate these optimized designs, we fabricated them using a 3D printing method (Fig. 4b–d). The geometric fidelity of the printed samples was first confirmed via 3D scanning, revealing dimensional deviation within 10% of the computational models (Fig. 4e–g). For comparison, we also fabricated typical cuboidal TEGs with the same material volume. To evaluate performance, each device was placed between a heater and a water cooler inside a vacuum chamber (Supplementary Fig. 9), where the Q_h , temperatures, and output power were measured.

In all samples, the measured power generation performance closely matched computational predictions for the optimized geometries (Fig. 4h–n). Specifically, BiSbTe achieved a peak η of 5.2%, more than 2.5 times higher than the cuboid’s 2%. Likewise, PbTe and Cu₂Se reached peak η values of 5.45% and 5.14%, exceeding their respective cuboid efficiencies by factors of 8.2 and 2. Furthermore, the observed η enhancements at the optimized operating point aligned well with computational predictions, showing increases of 186% for BiSbTe, 626% for PbTe, and 332% for Cu₂Se. These significant improvements arose from several factors, including increased R_{th} and the resulting increase in $\mathcal{E}$, optimized contact areas for fine-tuning R_{el} , and adjustments in R_m . Additional details on measured T and Q_h are provided in Extended Fig. 9. To further validate our approach, we also examined performance under varying Q_h and T (Supplementary Fig. 10), and in every case, the experimental results for the optimized geometries aligned with simulation predictions, consistently outperforming the conventional cuboid design.

Discussion

Our study demonstrates that TO can systematically generate optimal geometries for TE materials, maximizing power generation efficiency under diverse boundary conditions and material properties. This approach reveals universal design strategies for enhancing η . Under varying thermal conditions, TO consistently produces distinct geometric features, including a pronounced I-shape or an asymmetric hourglass structure, by balancing the conversion factor and matching conductance. Furthermore, intrinsic material properties interact with geometry, influencing efficiency gains. Materials with steep temperature-dependent properties exhibit significant η enhancements through geometric adaptation, whereas those with inherently low κ yield only marginal improvements. Experimental validation using 3D-printed materials confirms the feasibility and effectiveness of our computationally derived designs. These findings emphasize the fundamental role of geometry in reconciling imbalances between electrical and thermal transport properties induced by material characteristics. The scalability of our TO framework—applicable across diverse boundary conditions, heterogeneous materials, and multi-objective optimization problems—suggests its potential for revolutionizing high-efficiency TE devices. Ultimately, this approach can drive advancements in a broad range of energy-harvesting technologies.

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Methods

Materials

Elemental granules of Bi, Sb, Se, and Te (99.999%) were purchased from 5 N Plus. Elemental Pb (99.95%) and Cu (99.9%) were purchased from Alfa Aesar. Elemental Na (>95%) was purchased from Aldrich Chemical. Isopropanol (99.5%) and Se (99.999%) powder for the synthesis of Se polyanion from Samchun Chemicals and Alfa Aesar, respectively. Ethanethiol (> 97%), ethylenediamine (> 99.5%), acetonitrile (> 99.8%), and glycerol (> 99.5%) were purchased from Aldrich Chemical.

Topology optimization

We employ a density-based TO approach to systematically design TEGs with enhanced power generation efficiency. This method redistributes material within a predefined design domain to optimize performance. The design problem is formulated as a mathematical optimization problem, maximizing either power generation (P) or η for a given constraint:

$$\max f(V, T; \boldsymbol{\rho}) = P(V, T; \boldsymbol{\rho}) \text{ or } \eta(V, T; \boldsymbol{\rho}) \quad (1)$$

subject to

$$V_{TE} \leq V_c$$

$$0 \leq \rho_i \leq 1 \quad (i = 1 \dots n) \quad (2)$$

$$\mathbf{R}(V, T) = \mathbf{0}$$

Where the state variables $V(\Omega)$ and $T(\Omega)$ are electric potential and temperature field defined in the design domain Ω , respectively. V_{TE} corresponds to the volume of material used,

and V_c represents the maximum allowable volume. $\mathbf{R}$ represents residual of the governing equation formulated via Galerkin method. These are solved by finite element analysis utilizing quadratic Lagrange hexahedron elements for accurate spatial resolution. The ρ_i is a mathematical design variable ranging from 0 to 1 to represent material existence within the element of index i . The objective function $f(\rho)$, either P or η , reads:

$$P = \int_{\Omega_l} \mathbf{J} \cdot \mathbf{E} d\Omega, \quad \eta = \frac{P}{Q_h} = \frac{\int_{\Omega_l} \mathbf{J} \cdot \mathbf{E} d\Omega}{\int_{\Omega_{TE}} \mathbf{q} \cdot \mathbf{n} d\Omega} \quad (3)$$

The governing physics of the TO framework couples thermal and electrical transport equations, which describe steady-state heat conduction, electrical potential distribution, and thermoelectric effects:

$$\nabla \cdot \mathbf{q} + \mathbf{J} \cdot \mathbf{E} = 0 \quad (4)$$

$$\nabla \cdot \mathbf{J} = 0$$

where $\mathbf{q}$ is the heat flux, $\mathbf{J}$ is the electric current density, and $\mathbf{E}$ is the electric field intensity. These equations are coupled through the Seebeck effect, which relates the generated voltage to the temperature gradient:

$$\mathbf{J}(\mathbf{V}, \mathbf{T}, \mathbf{x}) = \sigma(\rho)(\mathbf{E} - S(\rho)\nabla T) \quad (5)$$

$$\mathbf{q}(\mathbf{V}, \mathbf{T}, \mathbf{x}) = \alpha(\rho)T\mathbf{J} - \kappa(\rho)\nabla T$$

To represent the material properties with varying density ρ , we utilize the Solid Isotropic Material with Penalization (SIMP) method that is of wide use in engineering applications due to its computational efficiency and ease of implementation^{46,47}:

$$\Xi(x\rho) = \Xi_0(x_\epsilon + (1 - x_\epsilon)\rho^p)$$

where Ξ represents material property (i.e., σ, S, κ) and Ξ_0 is the intrinsic material properties

measured through experiments, respectively. The ρ_ϵ is small real value that prevents numerical instability, and p is the penalty parameter.

To improve computational stability and manufacturability, numerical techniques such as the Helmholtz filter and hyperbolic tangent projection with parameter continuation are applied to control minimum feature sizes and avoid checkerboarding artifacts^{48,49}. The optimization process is iteratively solved using the adjoint sensitivity method and the Method of Moving Asymptotes (MMA), ensuring convergence toward optimal solutions⁵⁰.

Upon convergence, the optimized material distribution is post-processed as a volumetric field, and a solid-void interface is extracted at a 50% density threshold to generate physically realizable geometries suitable for fabrication. R_{el} , ΔT , output voltage, output power, and η of the optimized and reference TE generator were computed based on the final optimized design, accounting for the influence of material density variations. All numerical procedures, including finite element analysis, sensitivity analysis, and optimization processes, were executed within the commercial software COMSOL Multiphysics, which was complemented by Matlab Livelink for the post-process and the sensitivity filtering.

TEG models

To systematically evaluate topology optimization across different system parameters, including thermal boundary conditions, material properties, and contact resistance, a standardized TEG model is established as shown in Fig. 1(a).

Each TEG consists of a TE material positioned between two copper electrodes. Initially, the TE region is designed as a cuboid measuring $6 \times 6 \times 6 \text{ mm}^3$, while the copper electrodes are

$6 \times 6 \times 3 \text{ mm}^3$. The TE domain is discretized into approximately 8,000 finite elements to ensure high-resolution optimization.

The boundary conditions for the TEG models are carefully defined to reflect realistic operational environments. All side surfaces are treated as adiabatic and electrically insulated. The top electrode functions as the hot-side with an electrical ground, while the bottom electrode serves as the cold-side, connected to an external load resistance of R_L to facilitate power generation. Additionally, contact resistivity at the TE-electrode interface is incorporated into the model, with values obtained from experimental measurements.

To assess the effects of different thermal boundary conditions, TO is conducted using BiSbTe properties under a range of thermal environments. The first set of simulations examines convective heating and cooling, varying the h from 10 to $1000 \text{ W m}^{-2} \text{ K}^{-1}$, while the ambient temperatures are fixed to 450 K on the hot side and 293.15 K on the cold side. Another set of scenarios involves applying a fixed heat flux to the hot side, with heat flux values of 5000, 7500, and 10000 W m^{-2} , while maintaining convective cooling on the cold side (Supplementary Fig. 3). In addition, a series of optimizations explore conditions where the hot-side temperature is held constant, and varying cooling fluxes are applied to ensure the device operates within the thermal stability range of the materials. To ensure the material operates within the valid range, a minimum temperature of 323.15 K was enforced to ensure realistic operating conditions for the TE device (Supplementary Fig.4).

The influence of material properties is also evaluated by applying topology optimization to nine different thermoelectric materials, including BiSbTe, PbTe, Cu_2Se , and FeNbSb. These materials are selected to cover a broad range of temperature-dependent electrical and thermal characteristics. The boundary conditions were set such that the hot side received a constant heat

flux of $10,000 \text{ W m}^{-2}$ while convective cooling was applied with a convection coefficient of $1200 \text{ W m}^{-2} \text{ K}^{-1}$ and an ambient temperature of 293.15 K . The optimizations with virtual materials were conducted under the same system conditions.

Additionally, to perform optimisations examining variations in contact resistance, aspect ratio, and load resistance, a common boundary condition was adopted: a constant heat flux of 5000 W m^{-2} was supplied from the hot side, with convective cooling characterized by a convection coefficient of $1200 \text{ W m}^{-2} \text{ K}^{-1}$ and an ambient temperature of 293.15 K . The optimal design according to the objective function was performed under fixed conditions of hot side temperature of 450 K and cold side temperature of 323.15 K .

In the integrated design, a constant heat flux was supplied according to each material's properties. Specifically, a heat flux of 5000 W m^{-2} was applied for BiSbTe, 50000 W m^{-2} for PbTe and 15000 W m^{-2} for Cu_2Se . In all cases, convective cooling was maintained with a convection coefficient of $1200 \text{ W m}^{-2} \text{ K}^{-1}$ and an ambient temperature of 293.15 K . Additionally, the maximum hot-side temperatures were set to 750 K for PbTe and 800 K for Cu_2Se . The contact resistance was assigned based on the measured values at the interface between each material and the electrodes (Supplementary Fig. 1).

Synthesis of viscoelastic TE inks

TE powders for both BiSbTe and Na-doped PbTe were synthesized using the same high-energy ball milling process with a Monomill (Pulverisette 6, Fritsch) for 11 h. An 80 ml zirconia milling jar with 5 mm zirconia grinding balls was used at a ball-to-powder weight ratio of 5:1. Milling for both BiSbTe ($\text{Bi}_{0.55}\text{Sb}_{1.45}\text{Te}_3$) and Na-doped PbTe ($\text{Pb}_{0.98}\text{Na}_{0.02}\text{Te}$) was conducted at 450 rpm. To synthesize Cu_2Se , elemental Cu and Se powders were ball-milled in a ball miller (8000M Mixer/Mill, SPEX) for 200 min and subsequently sieved to remove particles larger

than 45 μm . For the BiSbTe ink, a Sb_2Te_3 ChaM binder was synthesized by dissolving 0.64 g of antimony powder and 1.36 g of tellurium powder in a co-solvent comprising 4 mL of ethanethiol and 16 mL of ethylenediamine in a nitrogen-filled glove box. After stirring for over 24 h until the solution turned dark purple, 40 mL of acetonitrile was added to precipitate the binder via centrifugation at 7,500 rpm for 10 min, and the precipitate was then dried under vacuum for 30 min. This binder was mixed with 4 g of BiSbTe powder and 4 g of glycerol using a planetary centrifugal mixer (ARM-100, Thinky) for 2 h to yield a homogeneous ink. For the PbTe ink, the TE powder was combined with 0.5 wt% tellurium powder and dispersed in glycerol, with the glycerol mass set to half that of the total powder. The mixture was then homogenized for 1 h in the planetary centrifugal mixer. For the Cu_2Se ink, a Se polyanion additive was first prepared by dissolving 0.5 g of selenium powder in 0.5 mL of ethanethiol and 4.5 mL of ethylenediamine at room temperature with vigorous stirring for 72 h. Five millilitres of this solution were mixed with 37.5 mL of isopropanol and centrifuged at 7,715 g for 10 min to precipitate the Se polyanion, which was then vacuum-dried for 1 h. Finally, 2–3 g of ball-milled Cu_2Se powder and 1–1.5 g of the dried Se polyanion were dispersed in 2.5 g of glycerol and mixed for 2 h in the planetary centrifugal mixer to produce the final ink.

3D printing of TE materials

3D printing was performed using a home-built, extrusion-based printer equipped with programmable pressure controls. The synthesized ink was loaded into a 5 ml syringe (Saejong) fitted with a metal nozzle having an inner diameter of 340 μm . The printing process was carried out at room temperature with a 1 s interval between layer depositions. The as-printed sample was then dried at 423 K for over 12 h and subsequently annealed under a nitrogen atmosphere under different conditions: BiSbTe was annealed at 723 K for 2 h; PbTe underwent a two-step

annealing at 1073 K for 2 h followed by 923 K for 3 h; and Cu₂Se was annealed at 1233 K for 13 h.

3D scanning of TE materials

Conformity between the 3D-printed samples and the design drawings was assessed using a 3D scanner (Gom Scan, Zeiss) employing a non-contact optical measurement technique (fringe projection with a blue LED). The resulting STL files were analyzed for congruence using the commercial software Control X.

Fabrication and power measurement of TE device

The printed samples were attached to Cu electrodes using SAC solder paste at 250 °C for BiSbTe, Ni–Sn paste at 450 °C for PbTe, and silver paste (Pyro-Duct 597-A, Aremco) for Cu₂Se. Output power was measured under a controlled temperature difference using a ceramic heater as the heat source and a water-circulating chiller for cooling, while K-type thermocouples connected to a Keithley 2000 multimeter monitored the temperature changes. To prevent unintended oxidation and air convection, all evaluations were performed within a vacuum chamber.

Method reference

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Data availability

All data generated or analysed during this study are included in the published article and Supplementary Information. The source data underlying the figures of the Main Text are provided within the “Source Data” file.

Code availability

The COMSOL code generated for the current study is available from the corresponding authors on reasonable request.

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Author contributions

J.L., S.E.Y., and S.C. contributed equally to this work. J.L, S.E.Y., S.C., H.C., and J.S.S. designed the experiments, analysed the data, and wrote the paper. J.L., H.L., and H.C. performed the topology optimization studies. J.L., S.E.Y., S.C., H.H., K.K., Y.E.P., and carried out the fabrication and measurement of the devices. All authors discussed the results and commented on the manuscript.

Competing interests

The authors declare no competing interests.

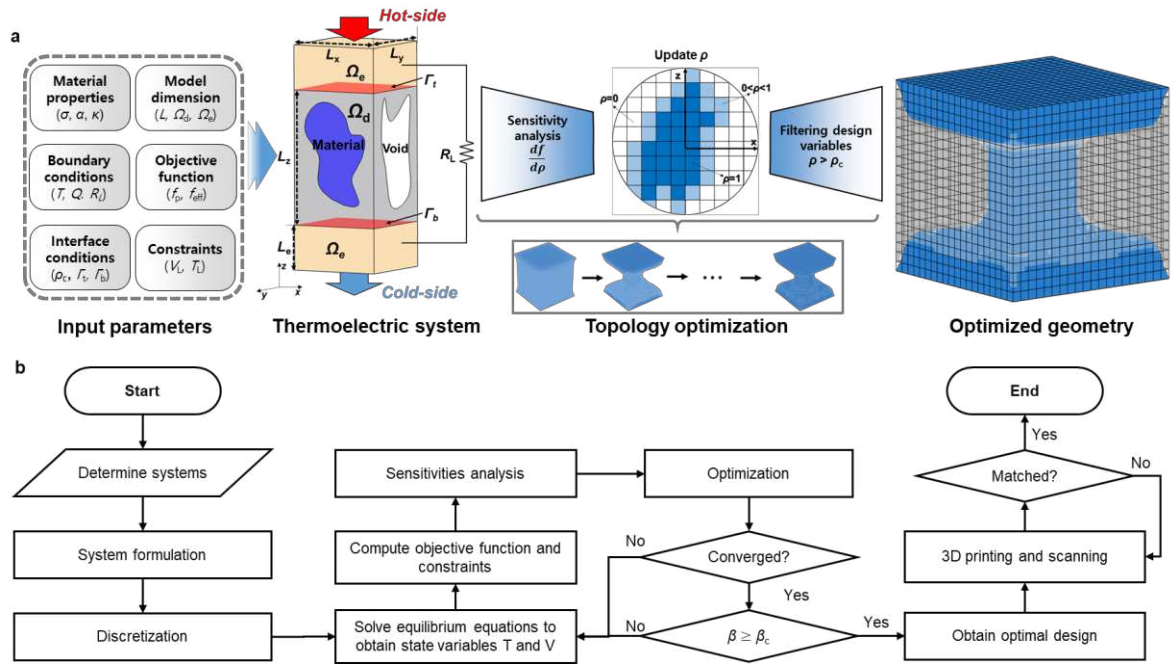


Fig. 1 | Overview of the TO process for TE materials. **a**, Schematic depiction of the TO process, illustrating how the optimal layout of TE materials is determined under various system parameters. The layout is optimized by maximizing the objective function (P or η) defined within the TE power generation system. **b**, Flowchart summarizing the sequential steps of the TO process for TE materials. In this flowchart, T represents temperature, V denotes voltage, β indicates the projection slope, and β_c is the predefined threshold value.

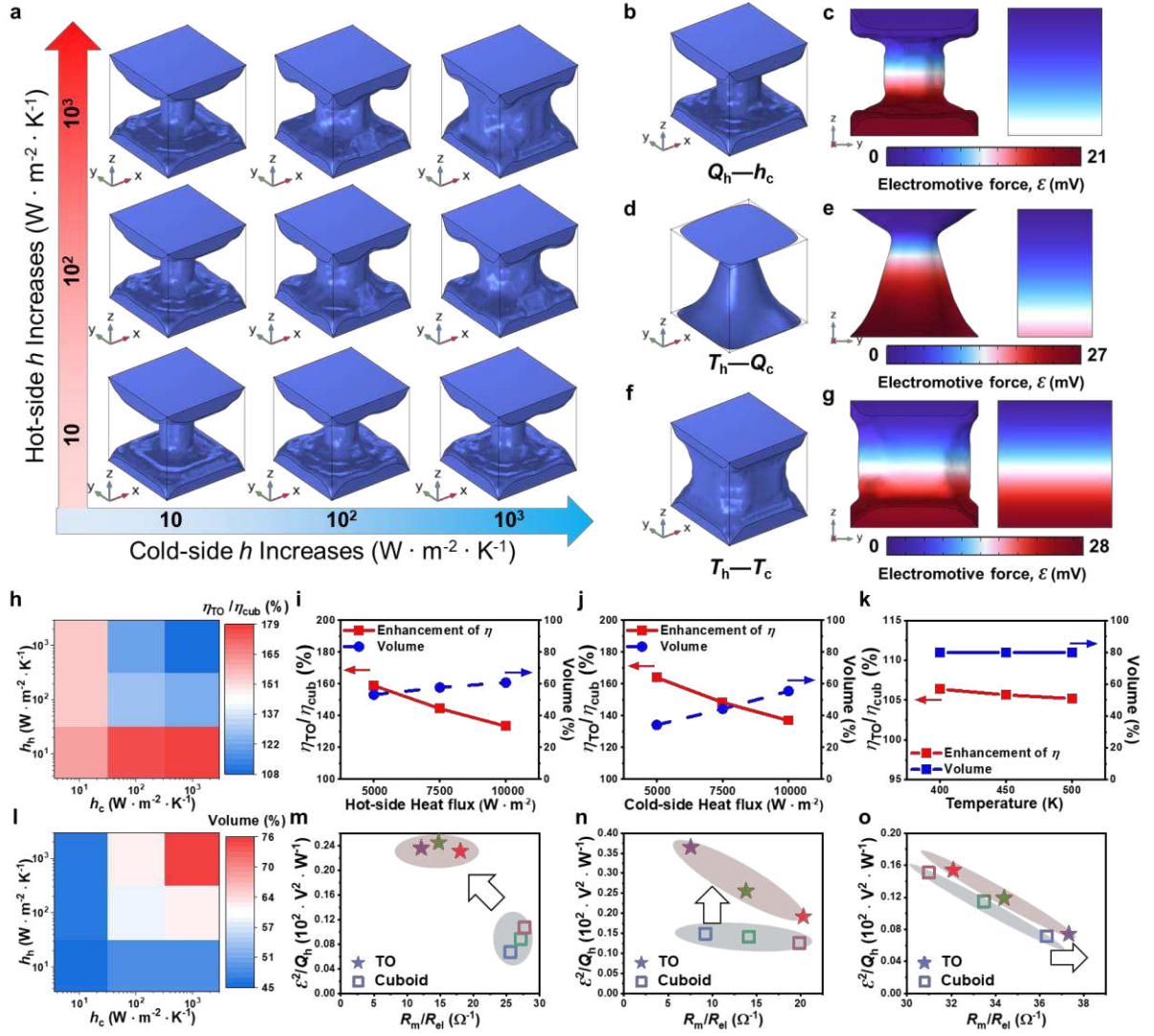


Fig. 2 | Optimums with different thermal boundaries. **a,b,d,f**, Optimized BiSbTe geometries for **(a)** convective cooling and heating conditions with h of $10 - 10^3 \text{ W m}^{-2} \text{ K}^{-1}$; **(b)** fixed Q_h and convective cooling; **(d)** fixed T_h and Q_c ; and **(f)** fixed temperature boundaries on both sides. **c,e,g**, Comparison of the electromotive force distributions for each optimized geometry (left) and a reference cuboid with the same material volume (right), corresponding to **b** (**c**), **d** (**e**), and **f** (**g**), respectively. **h-l**, η enhancement compared to a reference cuboid of the same volume, together with the volume fraction of the optimized geometry as a percentage of the total design domain. Results from **a** are shown in **h**, **l**, from **b** in **i**, from **d** in **j**, and from **f** in **k**. **m-o**, Analysis of the η enhancement achieved through optimal design from **b** (**m**), from **c** (**n**), from **d** (**o**). Blue represents the lowest heat or temperature, green intermediate values, and red the highest, whereas star symbols mark the optimized designs and hollow squares denote the reference cuboid. $\mathcal{E}^2/Q_h$ indicates the conversion factor, and R_m/R_{el} denotes the matching conductance.

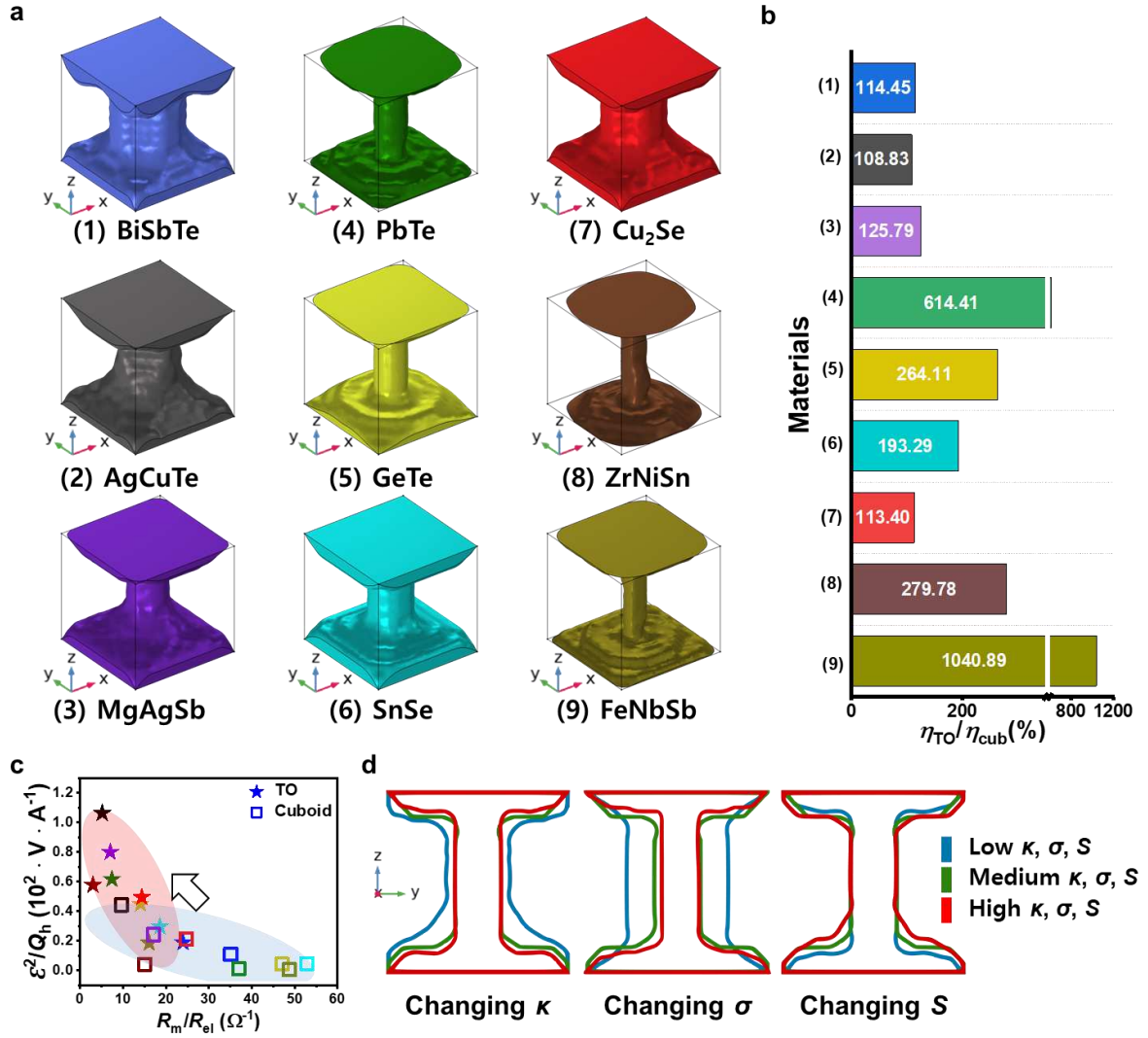


Fig. 3 | Optimums with different materials. **a**, Optimized geometries derived from TO employing nine distinct sets of material properties. **b**, η enhancement relative to a cuboid of equivalent material volume from **a**. **c**, Analysis of η enhancement for the optimal designs from **a**. Star symbols denote the optimized designs, hollow squares denote the reference cuboid, and the colors (in nine variations) match those in **a**, indicating the corresponding set of material properties. **d**, Side-view outlines of the optimized geometry, representing its shape as seen from the side, under a virtual material model in which k , σ , and S are varied. The baseline properties are $k = 2 \text{ W m}^{-1} \text{ K}^{-1}$, $\sigma = 1000 \text{ S cm}^{-1}$, and $S = 130 \text{ } \mu\text{V K}^{-1}$, with $k = 0.5, 2, 5 \text{ W m}^{-1} \text{ K}^{-1}$; $\sigma = 100, 1000, 5000 \text{ S cm}^{-1}$; and $S = 30, 130, 230 \text{ } \mu\text{V K}^{-1}$ shown in blue, green, and red, respectively.

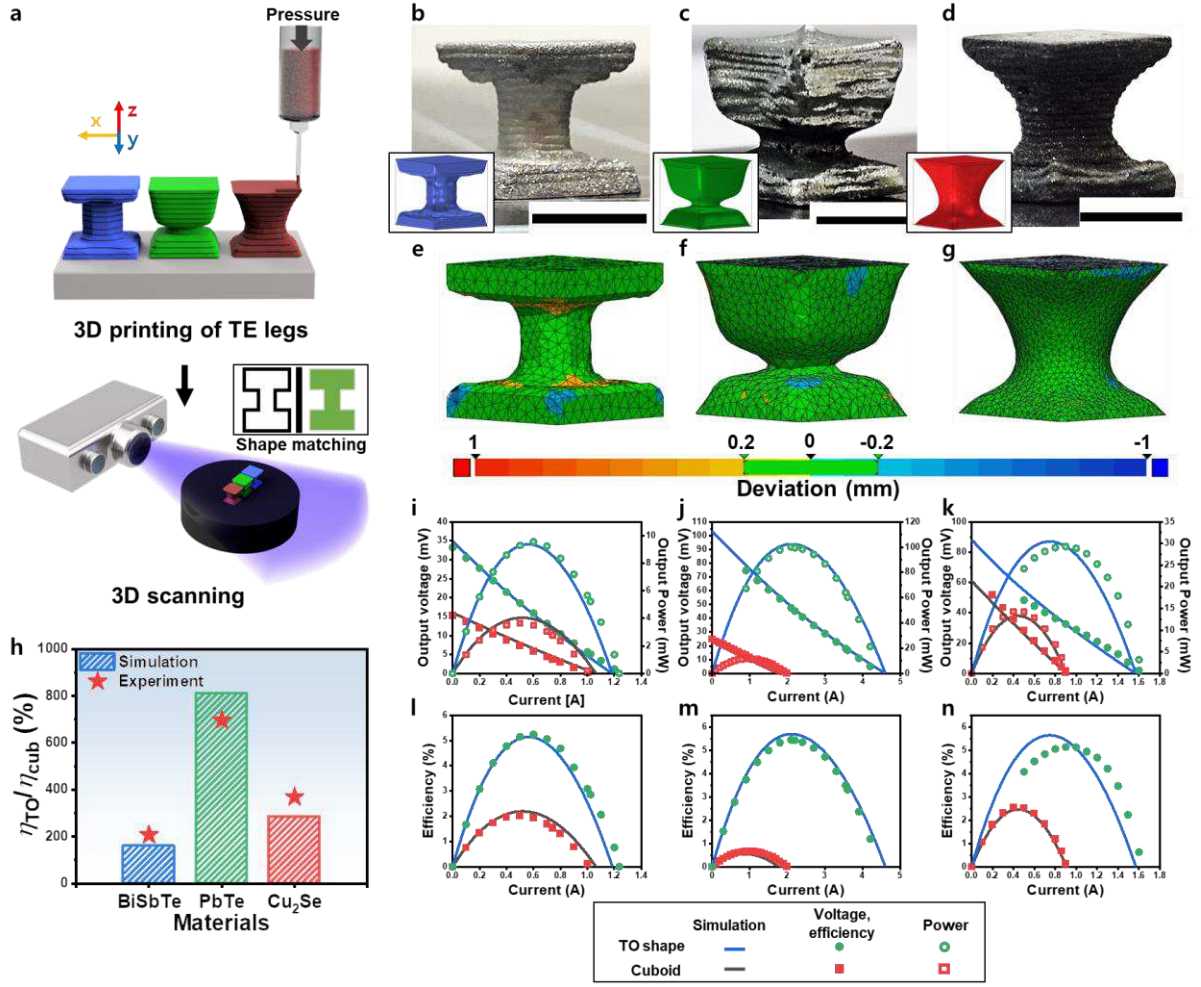


Fig. 4 | Integrated optimization and experimental validation. **a**, Schematic illustrating the 3D printing and scanning processes. **b–d**, Photographs of 3D-printed samples of BiSbTe (**b**), PbTe (**c**), and Cu₂Se (**d**), each featuring an optimized geometry derived from integrated TO. The scale bar is 5 mm. The insets in each panel show the corresponding optimized geometry. **e–g**, 3D scan images corresponding to **b** (**e**), **c** (**f**), and **d** (**g**), respectively. **h**, Measured and computationally predicted enhancements in η at the optimized operating condition for each sample, compared with a cuboid of the same material volume. **i–n** Measured voltage and power (**i–k**) and η (**l–n**) as a function of current for BiSbTe (**i, l**), PbTe (**j, m**), and Cu₂Se (**k, n**).

Supplementary Files

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